MATH 211 Fall 2015 **TEST I**

NAME: **SECTION:**

(1) True/False Questions (2 points each): Here A, B etc. denote sets while $x, y, \cdots$ denote elements, and $\mathcal{P}(A)$ denotes the power set of a set A

T **F** The logical expression $\exists x P(x) \rightarrow Q(x)$ is a proposition.

T **F** Universal generalization states that if $\forall x P(x)$ is true, then you can prove that $P(c)$ is true for an arbitrarily selected member in the domain.

T **F** Resolution uses the tautology $(p \vee q) \vee (\neg p \vee r) \rightarrow (q \vee r)$.

T **F** Let $f : A \rightarrow B$ be an arbitrary function and S_1, S_2 be two subsets of B. Then $f^{-1}(S_1 \cap S_2) = f^{-1}(S_1) \cap f^{-1}(S_2)$.

T **F** $(A \cap \bar{B}) \cup (A \cap B) = A$

T **F** The set $\bigcap_{n=0}^{\infty} \{n, n+1, n+2, \cdots\}$ is infinite.

T **F** There exist two uncountable sets A and B such that $A - B$ is countably infinite.

T **F** It was proven in class that the number $\sqrt{2}^{\sqrt{2}}$ was irrational.

T **F** Although the two numbers $1.999\cdots$ and 2 are very close, they are essentially two distinct numbers.

T **F** The converse of the proposition "You don't need an umbrella unless it is not sunny" is: "If it is not sunny you need an umbrella".

T **F** If $A = \{a, b\}$, $B = \{c, d\}$, then $(\{a\}, \{c\})$ is a member of $\mathcal{P}(A) \times \mathcal{P}(B)$.

T **F** The set $\mathcal{P}(\mathcal{P}(\phi))$ has exactly two elements.

(2) In the following use the following predicates. We define the predicates: $E(x)$: "x is an employee"; $T(y)$ is "y is a task", $R(x, y)$: "x is trained to do y", $P(x, y)$: "x performs y". Translate the following statements from logical expressions to English or vice versa, as appropriate.: (5 points each)

(a) All employees perform exactly the tasks that they are trained to do.

(b) There are two employees who are not trained to do the same task, but any task is performed by one of them.

(c) $\forall x \exists y (E(x) \rightarrow (T(y) \wedge \neg R(x, y))) \vee \exists x_1 \forall y (E(x_1) \wedge (T(y) \rightarrow (R(x_1, y) \wedge \forall x_2 (E(x_2) \wedge R(x_2, y) \rightarrow x_1 = x_2)))$

(d) $\forall x_1 \forall x_2 \forall y ((E(x_1) \wedge E(x_2) \wedge T(y)) \rightarrow ((x_1 = x_2) \vee (R(x_1, y) \rightarrow \neg R(x_2, y))))$

(3) (8 pt.) Simplify the following logical expression as much as possible, without using a truth table. Make sure you name the laws of inference that you use:

$(p \vee q) \wedge (\neg p \vee r) \wedge (q \rightarrow r)$.

(4) (8 pt.) Show that $|[0, 1)| = |(0, 1)|$. That is, show that the two sets sets have the same cardinality.

(5) (8 pt.) An integer is a perfect square if it can be written as k^2 for some integer k. prove or disprove that $3 \times 10^{450} + 7$ is a perfect square.

(6) (8 pt.) Let f be a function from a set A to a set B, and let S be a subset of A. Find a condition that implies that $f^{-1}(f(S)) = S$. Prove your statements.

(7) Show that $\sqrt[4]{2}$ is irrational.